SECURITY CLASSIFICATION OF THIS PAGE **DTIC FILE COPY**

REPORT DOCUMENTATION PAGE

AD-A201 270

1b. RESTRICTIVE MARKINGS

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release; distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

5. MONITORING ORGANIZATION REPORT NUMBER(S)
AFOSR-TR- 88-1189

6a. NAME OF PERFORMING ORGANIZATION
University of Wisconsin-Madison

6b. OFFICE SYMBOL (If applicable)

7a. NAME OF MONITORING ORGANIZATION
Air Force Office of Scientific Research

6c. ADDRESS (City, State and ZIP Code)
Center for the Mathematical Sciences
610 Walnut Street - WARF Bldg.
Madison, WI 53705

7b. ADDRESS (City, State and ZIP Code) Bld 410
Directorate of Mathematical & Information Sciences, Bolling AFB, DC 20332-6448

8a. NAME OF FUNDING/SPONSORING ORGANIZATION
AFOSR

8b. OFFICE SYMBOL (If applicable)
NM

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR-87-0191

8c. ADDRESS (City, State and ZIP Code)
Bld 410
Bolling AFB, DC 20332-6448

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
61102F	2304	A9	

11. TITLE (Include Security Classification) Interdisciplinary Research in Viscoelasticity & Rheology

12. PERSONAL AUTHOR(S)
D. S. Malkus and J. A. Nohel

13a. TYPE OF REPORT
Annual Technical

13b. TIME COVERED
FROM 5/01/87 TO 5/31/88

14. DATE OF REPORT (Yr., Mo., Day)
6/22/88

15. PAGE COUNT
8

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB. GR.

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Keywords: Viscoelastic materials, shear flow, stability, finite difference & finite element methods, systems of hyperbolic conservation laws.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Abstract. Viscoelastic materials with fading memory exhibit behavior that is intermediate between the nonlinear hyperbolic response of purely elastic solids and the strongly diffusive, parabolic response of viscous fluids. The primary objective of current research is the modeling, analysis and computation of unsteady motions of viscoelastic materials with fading memory. During the first year of funding, progress has been made in: (1) Understanding 'spurt' phenomena occurring in shearing flows of viscoelastic fluids; computational results for the unsteady equations produce qualitative and quantitative agreement with careful experimental results. (2) Understanding of weak solutions which are sufficiently broad to include shocks and acceleration waves. In (1), significant progress is being realized through insight gained from an interplay between careful numerical experiments and analysis.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL
Dr. Arje Nachman

22b. TELEPHONE NUMBER (Include Area Code)
(202) 727-5027

22c. OFFICE SYMBOL
NM

DD FORM 1473, 83 APR — EDITION OF 1 JAN 73 IS OBSOLETE.

SECURITY CLASSIFICATION OF THIS PAGE

INTERDISCIPLINARY RESEARCH IN VISCOELASTICITY & RHEOLOGY

David S. Malkus & John A. Nohel

with

Robert C. Rogers, Athanassios E. Tzavaras & Students

Research Objectives

Viscoelastic materials with fading memory, e.g. polymers, suspensions, emulsions, exhibit behavior that is intermediate between the nonlinear hyperbolic response of purely elastic materials and the strongly diffusive, parabolic response of viscous fluids. They incorporate a subtle dissipative mechanism induced by effects of the fading memory. The understanding of the equations of motion coupled with various constitutive assumptions at the mathematical level is crucial for modelling, design of algorithms and computation of particular problems. During the first year of funding, we have placed a major emphasis on the study of shear flows of viscoelastic fluids. Such flows exhibit a variety of interesting physical phenomena of importance, for example, in polymer processing. We have been intrigued by the fact that many numerical methods used in the computation of (supposedly steady) viscoelastic fluid flows appear to fail in physically relevant regions of parameter space and thus do not capture important phenomena. In order to gain a deeper understanding of such problems, we have recently initiated a study involving modeling, analysis and computation of "spurt" phenomena, the occurrence of which in certain shear flows of non-Newtonian fluids has been confirmed by careful experiments in capillary rheometry. This is the first of several problems that we proposed to study during the funding period, and it has proved to be of surprising physical and mathematical interest. Our overall goals in dealing with this problem and others to which we will turn our attention to in the future are:

1. To understand qualitative properties of the mathematical model: the global existence and uniqueness of solutions, dependence on data, regularity and asymptotic behavior of solutions for large time, approach to steady states, etc.
2. To understand the physical consequences of the model. Do any or all of the possible solutions make physical sense? Do solutions that have mathematically interesting character correspond to observed phenomena? Do they predict behavior that should be studied in the laboratory? What solutions to the problem are relevant to processing and design?
3. To understand the physical model. How do the observed solutions correspond to the molecular or continuum model on which they are based? Can the character of these solutions serve to validate the physical model or suggest improvements in it?
4. To design numerical methods that account for the mathematics and reproduce the physics. The current focus is on one-dimensional model problems, but in the long-term, the goal is to adapt results for the model problems to flows in complex geometries.
5. To study the broad mathematical implications of 1. - 4. for these and related classes of problems. We note that concepts such as weak solutions, shocks, shear bands, phase changes, etc arise in a variety of problems of physical and technological importance.

Status of Research

To achieve these goals, our interdisciplinary program is adapting and extending tools in nonlinear analysis of partial differential equations, analytical and computational techniques for hyperbolic

conservation laws, and computational techniques from nonlinear structural dynamics. Awareness of the latest developments in the rheology of viscoelastic liquids is vital in these efforts.

Spurt in shear flows. The spurt phenomenon was discovered by Vinogradov, et al., (1972) in flow of rubber-like liquids (polyisoprenes) through small capillaries. It is characterized by a sudden, large increase in volumetric flow rate, which occurs at a fixed critical stress that is independent of the molecular weight of the liquid. This phenomenon had been overlooked or dismissed by rheologists because no plausible mechanism was known. It was lumped together with instabilities such as "slip," "apparent slip," and "melt fracture," which are poorly understood. While regarded as anomalous, these instabilities can severely disrupt polymer processes; they can be avoided in practice only with ad hoc engineering expedients. The mechanisms of such phenomena are not understood primarily because the governing equations are analytically intractable and because popular numerical methods fail to capture these and other dramatic non-Newtonian effects. We expect that our research will have practical merit if it can help to overcome these formidable hindrances to predictive modeling of processing flows.

We study a model initial boundary value problem for (incompressible) shear flow of a non-Newtonian fluid between parallel plates, driven by a constant pressure gradient. The equations of motion are derived from a fully three-dimensional, time-dependent problem for which the reduction to one dimension is a reasonable physical approximation (see [N2]). The components of the stress tensor are assumed to obey a particular differential constitutive assumption (due to Johnson and Segalman [N2]; an extension to more general constitutive laws characterizing materials with fading memory appears plausible). Three one-dimensional model viscoelastic flow problems, namely longitudinal compression of a viscoelastic solid with a nonlinear Boltzmann constitutive law, shearing flow of a viscoelastic fluid described by the Johnson-Segalman constitutive relations, and elongational flow of a Johnson-Segalman fluid, are being studied; the emphasis is on shearing flows. The evolution equations have been cast as conservation laws, and in each case they take the form of gas dynamics with relaxation terms. The analysis and design of numerical methods is based on adapting and extending techniques of nonlinear hyperbolic systems.

In the absence of a Newtonian viscosity contribution to the stress, it is shown in [N2, Ch.II] that the necessarily nonlinear equations of motion can change type (from strictly hyperbolic to elliptic), resulting in Hadamard instability and the loss of evolutionary character. For smooth initial data in the hyperbolic region, the method of characteristics shows that initially smooth solutions exhibit finite-time blow-up (of derivatives) if the initial data are sufficiently large (see [N2, Ch.II]; see also [N1] for a different application of the same method). Energy methods developed in [N2, Ch.IV] can be used to show that smooth solutions exist globally in time if the initial data lie in the hyperbolic region and are sufficiently small. In the presence of a Newtonian contribution to the total stress, the governing system of equations is evolutionary but cannot be classified according to type; Nohel and Tzavaras have established global time existence and are examining asymptotic behavior for large t. For the problem of physical interest, the Newtonian contribution to the stress is a small fraction of the polymer contribution (approximately 1.5%).

In joint work with B. J. Plohr [M6], we develop three numerical methods for the model initial boundary value problem: one based on Plohr's recent work on Riemann problems for hyperbolic conservation laws with a regularizing Newtonian viscosity, another based on using the total stress (rather than the velocity gradient) as a dependent variable in the balance of linear momentum, and a third based on appropriate modifications of an existing solid-mechanics code (for details see [M6]). Each of the three techniques yields satisfactory qualitative and quantitative agreement with Vinogradov's experimental results. The steady numerical solutions are achieved by wave propagation and wave interactions. The numerical results exhibit discontinuities in the steady

solution, and they indicate that the spurt phenomenon occurs at a critical value of the total stress (near the shear stress maximum of the steady equations) as the driving pressure gradient is raised from values slightly below to slightly above critical. The model also has a local minimum in steady shear stress, so that reversal of this process leads to a hysteresis effect whose magnitude is governed by the difference between the stress at the local maximum and that at the local minimum. It is found in [M6] that this behavior is identical to the "shape memory hysteresis" predicted analytically by Hunter and Slemrod (1983) for a much simpler and more tractable model. On the basis of our work, we are able to propose a set of laboratory experiments which would corroborate our model and lend credence to our identification of a spurt mechanism [M6].

The numerical spurt solutions contain internal layers with jumps in the shear rate across them, called "singular surfaces" by Hunter and Slemrod; they are analogous to acceleration waves. Based on numerical experiments, we find that methods that construct the steady solution directly from the steady equations cannot determine the thickness of the layer between the wall and the singular surface. This is because any stress between the local maximum and local minimum can be critical. Therefore the stationary problem is in general ill-posed, whereas the dynamic problem appears to be well-posed and appears to determine a unique steady solution *asymptotically in time*. *Furthermore*, we find that "pseudodynamic" methods, which treat stationary problems as parabolic problems in artificial time, are unstable [M6] with respect to solutions with a jump in the shear rate. Numerical difficulties encountered in flow problems in complex geometries at high Weissenberg number might be explained by phenomena similar to acceleration waves.

What follows is a brief summary of the progress of each Principal Investigator, the Postdoctoral Research Associates and a Research Assistant:

D.S. Malkus. Malkus' primary progress has been in the development of the numerical method arising from structural dynamics. He currently is working out the linearized stability analysis of the method for inclusion in [M6] and generalizing the method to be applicable to a wide variety of constitutive equations. His emphasis in this generalization is on those constitutive equations that arise from kinetic theories, such as elastic or rigid dumbbells with hydrodynamic interaction, finite extensibility, or anisotropic drag. The aim is to be able to relate the spurt phenomenon and other shear-flow solutions to molecular motion — molecular stretching and, if possible, orientation. He also implemented the numerical method based on total stress and has made a careful comparison of the results computed by the two methods. Another intriguing area being investigated by Malkus is the possible molecular interpretation of very short relaxation times with small associated viscosity. The shear stress maxima and minima result in wide gaps in the relaxation spectra (these can be inferred from Vinogradov's linear viscoelastic data); the gap in our model is between the Newtonian contribution and the single nonzero relaxation time. But the shear stress extrema need not be introduced by way of Newtonian viscosity: we have constructed models with extrema using the Johnson-Segalman model with two nonzero, widely differing relaxation times, but as with Hunter and Slemrod's analytical model, a very small amount of Newtonian viscosity is required to obtain globally defined dynamic numerical solutions. Therefore, we are considering the following proposition carefully: Vinogradov's spectral gap is surely caused by widely separated macromolecular processes, and there is no solvent present. Is there a molecular basis for the existence of processes with much shorter relaxation times, which mimic the Newtonian viscosity that numerical methods require? Or is the small amount of Newtonian viscosity required to obtain numerical solutions in the two relaxation-time model (10% of that for the single relaxation time model) always artificial?

Malkus is involved also in the refinement of existing techniques for computing steady solutions for viscoelastic fluids; he has spent several years developing such methods. Though they are often

limited by iterative convergence difficulties, there are constitutive equations, e.g., the Curtiss-Bird model for which this does not happen. Thus, there are many interesting problems that can be solved with the existing methods.

J.A. Nohel. The numerical results obtained thus far have motivated J.A. Nohel and A.E. Tzavaras to undertake a deeper analytical study of the unsteady shear-flow problem discussed above. By considering model problems which incorporate the key features of the above structure, they demonstrate the global existence of classical solutions for arbitrary smooth initial data, and they obtain results on asymptotic behavior for large time; in particular, it is shown that the total stress tends to zero in the time-asymptotic limit. The current effort is to obtain the limiting behavior of the polymer contribution to the stress and the limiting velocity profile. It may be possible also to establish analytical results on phase transitions, co-existence of phases, and hysteresis phenomena for this class of problems, extending results of Hunter and Slemrod (1983) and of Pego (1987) for simpler models. For the problem of shear flows based on the Johnson-Segalman constitutive law, and in the presence of any Newtonian viscosity contribution to the total stress, Nohel and Tzavaras have established the global-time existence of classical solutions for initial data of arbitrary size. Challenging issues must be overcome in order to obtain the asymptotic results.

Recent research has contributed significantly to the understanding of classical solutions of various initial value problems governing motions of viscoelastic materials with fading memory (see [N2]; see also [N5], which contains several improvements of earlier results). However, there are only very special results for weak solutions. For an important special case of a particular class of one-dimensional models described below, Nohel, Rogers & Tzavaras have recently used the method of vanishing viscosity and compensated compactness to establish the existence of global weak solutions (in the sense of distributions) for bounded measurable initial data of arbitrary size (see [N3] and [N4]); there is no satisfactory uniqueness result.

We consider the existence of global weak solutions, in the class of bounded measurable functions, for the Cauchy problem for the system

$$\begin{aligned} w_t &= v_x, \\ v_t &= \sigma_x, \end{aligned} \qquad x \in \mathbf{R},\ t > 0, \tag{VE}$$

with initial conditions

$$w(x,0) = w_0(x), \quad v(x,0) = v_0(x), \quad x \in \mathbf{R}. \tag{icc}$$

Here the function $\sigma(x,t)$ is determined by the history of $w(x,\cdot)$ through the constitutive assumption

$$\sigma(x,t) = \varphi(w(x,t)) + \int_0^t k(t-\tau)\psi(w(x,\tau))d\tau. \tag{CA}$$

The given functions $\varphi(w), \psi(w)$ and $k(t)$ are assumed to be smooth and, in addition,

$$\varphi'(w) > 0, \quad w \in \mathbf{R},$$

so that the structure of (VE) is hyperbolic. This equation is a widely accepted model for important phenomena in viscoelastic solids. The special case $\psi \equiv \varphi$, is considered when the data $w_0, v_0 \in L^\infty(\mathbf{R}) \cap L^2(\mathbf{R})$. The problem becomes

$$\begin{aligned} w_t &= v_x, \\ v_t &= \varphi(w)_x + \int_0^t k(t-\tau)\varphi(w(\cdot,\tau))_x d\tau, \qquad x \in \mathbf{R},\ t > 0, \\ w(x,0) &= w_0(x),\ v(x,0) = v_0(x), \quad x \in \mathbf{R}. \end{aligned} \tag{1.1}$$

The constitutive function φ is assumed to satisfy

$$\begin{cases} \varphi : \mathbf{R} \to \mathbf{R} \text{ is a twice continuously differentiable} \\ \text{function such that } \varphi'(w) > 0, \;\; w \in \mathbf{R}; \\ \varphi \text{ has a single inflection point at } w = w_i \text{ and is} \\ \text{convex on } (w_i, \infty) \text{ and concave on } (-\infty, w_i). \end{cases}$$

The kernel k satisfies

$$k : [0,\infty) \to \mathbf{R}, \quad k \in C^1[0,\infty),$$

and the data $w_0(x), v_0(x)$ satisfy

$$w_0(x),\; v_0(x) \in L^\infty(\mathbf{R}) \cap L^2(\mathbf{R}).$$

The following theorem was proved.

Theorem: *Under the above assumptions, given $T > 0$, there exists a weak solution $\{w(x,t), v(x,t)\}$ of (1.1) on $\mathbf{R} \times [0,T]$, such that*

$$(w,v) \in L^\infty([0,T]; L^2(\mathbf{R})) \cap L^\infty(\mathbf{R} \times [0,T]).$$

DiPerna established similar results in the case of 1-d elasticity and 1-d isentropic gas dynamics. Thus far, efforts to extend these results to more general models appear to meet difficulties very similar to those encountered when attempting to study weak solutions to conservation laws for nonisentropic gas dynamics. We intend to continue efforts in this direction.

R. C. Rogers. In addition to the joint work with Nohel and Tzavaras ([R5]) described above, Rogers persued research on self- effect problems in electro-magneto-elasticity ([R1], [R6]), modeling ferromagnetic materials ([R2], R7], [R8]), and problems in compensated compactness and weak continuity ([R3], [R4]).

A. E. Tzavaras. In addition to the joint work with Nohel and Rogers ([T3], [T4]) discussed above, Tzavaras studied the following problems.

Shear instabilities in the form of shear bands are often observed when metals are deformed at high strain rates. Since shear bands diminish the strength of materials and are often precursors to rupture, their understanding is critical for the development of improved materials. According to a popular theory, shear band formation is the result of an autocatastrophic process induced by thermal softening properties of materials. At high strain rates, non-uniform straining induces non-uniform heating which, in turn, enhances the plastic flow at hotter regions and reduces it at colder regions. This creates a destabilizing feedback mechanism to which there is opposition from internal dissipation and strain hardening.

In order to assess the contributions of the above factors, we have been studying the adiabatic plastic shearing of an infinite plate subjected to either steady shearing or prescribed tractions at the boundaries. For constitutive law we choose a parametric power law with parameters measuring the relative weight of thermal softening, strain hardening and strain rate sensitivity. The analysis of the above problem leads to the study of a system of partial differential equations consisting of a parabolic equation coupled through the diffusion coefficient with two equations of hyperbolic type. The main question is whether the solution stabilizes as $t \to \infty$ or whether nonuniformities develop and the material exhibits an unstable response. This question has been studied in [T1], [T6] with the following results. In case the shearing deformation is caused by prescribed tractions, the

parameter space (defined through the powers that enter in the constitutive law) can be decomposed into two subregions. In one of them the solutions are asymptotically attracted to a "uniform" dynamic solution, while in the complement instabilities develop. The analysis indicates that the latter are associated with a collapse of the ability of the material to diffuse the applied stress ([T6]). The case when the shearing deformation is caused by steady shearing at the boundary is more delicate; certain regions of stability have been identified, but the issue is still under investigation with emphasis on identifying the mechanism for the onset of instability in model problems.

In a joint program with M. Slemrod, Tzavaras is also involved in studying the wave admissibility criterion for the solution of the Riemann problem for hyperbolic conservation laws. It is well known that in solving the Riemann problem one encounters a severe loss of uniqueness which has to be accounted for by choosing an appropriate admissibility criterion. According to the wave admissibility criterion (introduced by Dafermos) the admissible solutions are chosen as limits of solutions of appropriate "viscosity regularized" problem, that are rigged so as to preserve the invariance of the equations under rescaling of the independent variables. This approach has been tested for the Riemann problem for the equations of gas dynamics in one space dimension in Eulerian coordinates [T5].

K. Barki, Ph.D. student. Mr. Barki's thesis research involves the development of new transient algorithms for finite element analysis of viscoplastic materials. The major focus is on a class of implicit/explicit methods with origins in structural dynamics, generalizing the method developed by Malkus to solve dynamic spurt. There are two fundamental ideas: first to allow implicit treatment of the elastic (highest) wave speed, which avoids severe stability limits; second, to avoid the need for stress interpolations by advancing the constitutive equations at Gauss-points in ODE fashion. Mr. Barki is focussing on problems in solid, 3-D viscoplasticity in a Lagrangian frame, but the ideas seem to be useful for fluid flow problems in an Eulerian frame. This idea is being pursued by Malkus. Mr. Barki has been using the CMS VAX system and the SDSC Cray XMP/48. He expects his Ph.D. in the summer of 1989.

Manuscripts Published, Submitted & in Preparation

D. S. Malkus:

M1. D. S. Malkus and M. F. Webster, On the accuracy of finite element and finite difference predictions of non-Newtonian slot pressures for a Maxwell fluid, *J. Non-Newtonian Fluid Mechanics* 25 (1987), 93–127.

M2. D. S. Malkus and X. Qiu, Divisor structure of finite element eigenproblems arising from negative and zero masses, *Comput. Meths. Appl. Mech. Engrg.* 66 (1988), 365–368.

M3. D. S. Malkus, M. E. Plesha, and M.-R. Liu, Reversed stability conditions in transient finite element analysis, *Comput. Meths. Appl. Mech. Engrg.* 68 (1988), 97–114.

M4. R. W. Kolkka, D. S. Malkus, M. G. Hansen, G. R. Ierley, and R. A. Worthing, Spurt phenomena of the Johnson Model fluid and related models, accepted for *J. Non-Newtonian Fluid Mechanics.*

M5. R. Cook, D. S. Malkus, and M. E. Plesha. Concepts and Applications of Finite Element Analysis, Third Edition, textbook manuscript accepted for publication by John Wiley and Sons, New York.

M6. D. S. Malkus, J. A. Nohel, and B. J. Plohr, Spurt phenomena in shear flows of non-Newtonian fluids – in preparation for the *Journal of Computational Physics.*

J. A. Nohel:

N1. J.A. Nohel and M. Renardy, Development of singularities in nonlinear viscoelasticity, Proceedings of Workshop on Amorphous Polymers, *IMA Volumes in Mathematics and its Applications*, Vol. 6, Springer-Verlag (1987), 139–152.

N2. M. Renardy, W.J. Hrusa, and J.A. Nohel, Mathematical Problems in Viscoelasticity, *Pitman Monographs and Surveys in Pure and Applied Mathematics*, vol. 35, Longman Scientific & Technical, Essex, England (1987), 273 pp.

N3. J.A. Nohel, R.C. Rogers, and A. Tzavaras, Weak solutions for a nonlinear system in viscoelasticity, *Comm. in P.D.E.* 13 (1988), 97–127.

N4. J.A. Nohel, R.C. Rogers, and A. Tzavaras, Hyperbolic conservation laws in viscoelasticity, *Proceedings of Conference on Volterra Integrodifferential Equations in Banach Spaces and Applications* held in Trento (Italy) February 1987, Longman Scientific & Technical (accepted).

N5. W.J. Hrusa, J.A. Nohel and M. Renardy, Initial value problems in viscoelasticity, invited paper, *Applied Mechanics Reviews* – submitted.

N6. D.S. Malkus, J.A. Nohel, and B.J. Plohr, Spurt phenomena in shear flows of non-Newtonian fluids – in preparation for the *Journal of Computational Physics*.

R. C. Rogers:

R1. Robert C. Rogers and Stuart S. Antman, Steady-state problems of nonlinear electro-magneto-thermo-elasticity. *Arch. Rat. Mech. Anal.* 95 (1986), 279–323.

R2. Robert C. Rogers, Nonlocal problems in electromagnetism, in Stuart S. Antman, J.L. Ericksen, David Kinderlehrer, and Ingo Müller, editors, *Metastability and Incompletely Posed Problems*, IMA, Springer-Verlag, New York (1986).

R3. Joel W. Robbins, Robert C. Rogers, and Blake Temple, On weak continuity and the Hodge decomposition, *Transactions of the AMS* 302 (1987).

R4. Robert C. Rogers and Blake Temple, A sufficient condition for weak continuity of polynomials in the method of compensated compactness. To appear in *Transactions of the AMS*.

R5. John A. Nohel, Robert C. Rogers, and A.E. Tzavaras, Weak solutions for a nonlinear system in viscoelasticity, *Communications in Partial Differential Equations*. 13 (1988).

R6. Robert C. Rogers, Nonlocal variational problems in nonlinear electro-magneto-elasticity. To appear in *SIAM J. of Math. Analysis*.

R7. Robert C. Rogers, A finite dimensional model problem for nonlocal exchange energy in ferromagnetic materials. Technical Report, Center for the Mathematical Sciences, University of Wisconsin-Madison (1988).

R8. Robert C. Rogers, A nonlocal model for the exchange energy in ferromagnetic materials. Technical Report, Center for the Mathematical Sciences, University of Wisconsin-Madison (1988).

A. E. Tzavaras:

T1. A. E. Tzavaras, Effect of thermal softening in shearing of strain rate dependent materials, *Arch. Rational Mech. and Anal.* 99 (1987), 349–374.

T2. A. Friedman and A. E. Tzavaras, Combustion in a porous medium, *SIAM J. Math. Anal.* 19 (1988), 509–519.

T3. J. A. Nohel, R. C. Rogers and A. E. Tzavaras. Weak solutions for a nonlinear system in viscoelasticity 13 (1988), 97–127.

T4. J. A. Nohel, R. C. Rogers and A. E. Tzavaras, Hyperbolic conservation laws in viscoelasticity, Proceedings of Conference on Volterra Integrodifferential Equations in Banach Spaces and Applications held in Trento (Italy) February 1987, Longman Scientific & Technical (accepted).

T5. M. Slemrod and A. E. Tzavaras, A viscosity approach for the solution of the Riemann problem in Eulerian gas dynamics - in preparation.

T6. A. E. Tzavaras, Interplay of thermal softening and strain rate sensitivity on the response of shearing motions - in preparation.

Professional Personnel

D. S. Malkus, Professor, Engineering Mechanics & Center for the Math. Sciences
J. A. Nohel, Professor, Mathematics & Center for the Math. Sciences
R. C. Rogers, Van Vleck Assistant Professor, Mathematics & Center for the Math. Sciences
A. E. Tzavaras, Van Vleck Assistant Professor, Mathematics & Center for the Math. Sciences
K. Barki, Ph. D. candidate, Engineering Mechanics

Interactions

The following presentations involving significant research interactions were given during the reporting period (we note that Nohel was unable to accept a number of invitations to national and international meetings):

D. S. Malkus, J. A. Nohel & B. J. Plohr, "Shearing flows of non-Newtonian fluids", Sixth Army Conference on Applied Math. and Computing, Boulder, CO, May 31, 1988. (Presented by Malkus)

J. A. Nohel, "Mathematical problems in viscoelasticity", Rheology Research Center, UW-Madison, March 4, 1988.

D. S. Malkus, J. A. Nohel & B. J. Plohr, "Instabilities in shear flows of viscoelastic fluids with fading memory", Seminar on PDE's and Continuum Models of Phase Transitions, Nice, France, 1/18-22/88.

D. S. Malkus, "New transient algorithms for non-Newtonian flows", Ill. Inst. of Tech., Math Department seminar, 1/26/88.

R. C. Rogers, "Nonlocal materials", MIPAC Seminar, UW-Madison, 12/2/87.

A. E. Tzavaras, Invited lecture based on joint work with Nohel and Rogers, Special Session on "Volterra Integral Equations", combined Midwest and Southeast Differential Equations Conference, Vanderbilt University, Nashville, TN, Oct. 1987.

A. E. Tzavaras, Invited lecture based on joint work with Nohel and Rogers, Special Session on "Qualitative Theory of Nonlinear PDE", 843rd. Meeting of the AMS, University of Maryland, College Park, MD, April 1988.

A. E. Tzavaras, "Formation of shear bands", LCDS Seminar in Applied Mathematics, Brown University, April 1988.

During the reporting period professional interactions (in some cases joint work) continued with B. J. Plohr (UW-Madison), W. J. Hrusa (CMU), L. Tartar (CMU), M. Slemrod (UW-Madison), C. M. Dafermos (Brown), R. Pego (U. of Michigan), M. Renardy (VPI), D. D. Joseph (U. of Minnesota), E. T. Olsen & B. Bernstein (IIT), T. J. R. Hughes (Stanford), and members of the Madison Rheology Research Center.